PERFORMANCE ANALYSIS OF FIR DIGITAL FILTER DESIGN TECHNIQUES

Prof. Gopal S. Gawande  
A.P., Department of electronics and telecommunication,  
S.S G.M College of Engineering, Shegaon (M.S.), INDIA

Dr. K.B. Khanchandani  
H.O.D., Department of electronics and telecommunication,  
S.S G.M College of Engineering, Shegaon (M.S.), INDIA

T.P. Marode  
M.E. Student  
S.S G.M College of Engineering, Shegaon (M.S.), INDIA

Abstract

The developments in electronic technology are taking place at mind blogging speed. Digital Signal Processing (DSP) is one of the fields where developments are taking place at a faster rate. The DSP applications demand high speed and low power digital filters. In order to meet these requirements, the order of the digital filter must be kept as small as possible. There are various sophisticated Computer Aided Design tools available to make the digital filter fast and power efficient. Filter design and analysis tool (FDA) is one of the Computer Aided Design tool available with MATLAB which enables design of the digital filter blocks faster and more accurate. In this paper the various FIR filter design techniques are analyzed for achieving minimum order.
I. Introduction

The developments in electronic technology are taking place at a tremendous speed. Recently, Digital Signal Processing (DSP) is used in numerous applications such as video compression, digital set-top box, cable modems, digital versatile disk, portable video systems/computers, digital audio, multimedia and wireless communications, digital radio, digital still and network cameras, speech processing, transmission systems, radar imaging, acoustic beam formers, global positioning systems, and biomedical signal processing. The field of DSP has always been driven by the advances in DSP applications and in scaled Very-Large-Scale-Integrated (VLSI) technologies.

Digital filters are an important class of Linear Time Invariant (LTI) DSP systems designed to modify the frequency characteristics of the input signal \( x(n) \) to meet certain specific design requirements. Digital filters are widely used because of certain advantages over Analog filters. Digital filters have the potential to attain much better signal to noise ratios than Analog filters. Digital Filters have emerged as a strong option for removing noise, shaping spectrum and minimizing inter-symbol interference (ISI) in communication architectures.[1]

Digital filters can be classified into Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) filters. The selection of FIR or IIR digital filter depends on the nature of the problem and specifications of the desired frequency response. In designing digital filters, it is usually desirable to have approximately flat frequency response, magnitude in pass band. Another important desirable feature of digital filter is of linear phase. A linear phase with integer slope corresponds to a simple delay in the time domain, and it reduces the phase distortion to a minimum in the frequency domain. The FIR filter of length M is described by the convolution of unit sample response \( h(n) \) of the system with input signal \( x(n) \) and is represented by the equation 1.[6]

\[
y(n) = \sum_{k=0}^{M-1} h(k) x(n-k)
\]

Thus impulse response of the filter denotes the coefficient of FIR filter.

FIR filters specification include the maximum tolerable pass band ripple, maximum tolerable stop band ripple, pass band edge frequency and stop band edge frequency.

Calculating coefficients of FIR filter requires considerable amount of computations. Therefore it is generally performed by using various Computer Aided Design tools such as Filter design and analysis (FDA) tool of MATLAB.
II. FIR filter Design Techniques

FIR filter design essentially consist of two parts:

i. Approximation problem

ii. Realization problem.

The approximation stage takes the specification and gives transfer function. Realization part deals with choosing the structure to implement the transfer function which may be in the form of circuit diagram or in the form of a program [3].

There are various methods to design FIR filter as follows:

I. Window technique.

II. Optimal filter design methods.

The windowing design technique is simple and convenient but not optimal i.e. order achieving is not minimum possible. Some of the windows commonly used are Blackman, Blackman-harris, Kaiser, Bohman, Chebyshev, Flat top, Gaussian, Hamming, Hann, Parzen, Rectangular, etc.

In frequency sampling technique for FIR filter design we specify the desired frequency response \( H_d(w) \) at a set of equally spaced frequencies at \( N \) samples. This method is useful for the design of non prototype filters where the desired magnitude response can take any irregular shape.

The various optimal filter design methods are Least square, Equiripple, Maximally flat, Generalized equiripple, Constrained band equiripple, etc. The basic idea in each method is to design the filter coefficients again and again until particular error is minimum [3].

III. Designing digital Filter using FDATool

The Filter Design and Analysis Tool (FDATool) is a powerful Graphical user interface (GUI) for designing and analyzing filters quickly. FDATool enables you to design digital FIR or IIR filters by setting filter specifications, by importing filters from your MATLAB workspace, or by adding, moving or deleting poles and zeros. FDATool also provides tools for analyzing filters, such as magnitude and phase response and pole-zero plots. FDATool seamlessly integrates additional functionality from other MathWorks products.

The specifications of FIR low pass filter are in given in table 1.

Table 1: FIR filter specifications.
In this paper, the various FIR filter design techniques are used to meet the specifications given in table 1 using FDA tool. The results of these FIR filter design techniques are then compared for minimal order.

### IV. Window Technique for digital FIR filter design

The simplest technique used to develop digital FIR filters is windowed filters. This technique is based on designing a filter using well-known frequency-domain transition functions called windows. The use of windows often involves choosing the lesser of two evils. Some windows, such as the rectangular, yield fast roll off in the frequency domain but have limited attenuation in the stop band along with poor group delay characteristics. Other windows, like the Blackman, have better stop-band attenuation and group delay but a wide transition band. Windowed filters are easy to use and scalable and can be computed on the fly by the DSP. This latter point means that a tunable filter can be designed with the only limitation on corner frequency resolution being the number of bits in the tuning word.[3]

In Window method, [4-6] from the desired frequency response specification $H_d(w)$, corresponding unit sample response $h_d(n)$ is determined using the following relation.

$$H_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(w)e^{jwn} dw \quad \text{equation 2}$$

where

$$H_d(w) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-jwn} \quad \text{equation 3}$$

Various types of windows can be used to meet the specifications given in table 1 and their relative performance is shown in table 2. While designing digital filter, order of filter (N) is always a matter of concern. In this paper, the performance of various FIR filter design methods is measured in terms of order required to implement digital filter.
Table 2. : Performance comparision of various windows for FIR filter design

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Type of Window</th>
<th>Passband attenuation(δp)</th>
<th>Stopband attenuation(δs)</th>
<th>Order of Window(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Blackman</td>
<td>2.492 dB</td>
<td>60.09026 dB</td>
<td>550</td>
</tr>
<tr>
<td>2</td>
<td>Blackman-Harris</td>
<td>2.5034 dB</td>
<td>64.62253 dB</td>
<td>700</td>
</tr>
<tr>
<td>3</td>
<td>Bohman</td>
<td>0.0001393 dB</td>
<td>60.1558 dB</td>
<td>900</td>
</tr>
<tr>
<td>4</td>
<td>Chebyshew</td>
<td>1.5769 dB</td>
<td>81.00921 dB</td>
<td>750</td>
</tr>
<tr>
<td>5</td>
<td>Flat top</td>
<td>1.1159 dB</td>
<td>109.3473 dB</td>
<td>1100</td>
</tr>
<tr>
<td>6</td>
<td>Gaussian</td>
<td>0.00222 dB</td>
<td>68.319 dB</td>
<td>850</td>
</tr>
<tr>
<td>7</td>
<td>Hamming</td>
<td>0.0016 dB</td>
<td>64.10399 dB</td>
<td>1250</td>
</tr>
<tr>
<td>8</td>
<td>Hann</td>
<td>0.0001880 dB</td>
<td>62.136 dB</td>
<td>800</td>
</tr>
<tr>
<td>9</td>
<td>Nutall</td>
<td>1.66671 dB</td>
<td>65.63894 dB</td>
<td>700</td>
</tr>
<tr>
<td>10</td>
<td>Parzen</td>
<td>2.2890 dB</td>
<td>73.64271 dB</td>
<td>1900</td>
</tr>
<tr>
<td>11</td>
<td>Rectangular</td>
<td>0.0006728 dB</td>
<td>66.4774 dB</td>
<td>1800</td>
</tr>
<tr>
<td>12</td>
<td>Kaiser</td>
<td>0.04369 dB</td>
<td>60.21 dB</td>
<td>160</td>
</tr>
</tbody>
</table>

The Hanning, Hamming and Blackman windows use progressively more complicated cosine functions to provide a smooth truncation of the ideal impulse response and a frequency response that looks better. But the best window results probably come from using the Kaiser window, which allows adjustment of the compromise between the overshoot reduction and transition region width spreading as shown in figure 1.
Figure 1. Magnitude response (dB) for Kaiser window.

Another window that closely approximates the desired frequency characteristics feed is Blackman window whose magnitude response is shown in figure 2.
V. Optimal Filter Design Methods for digital FIR filter design

In Optimal Filter Design Method various methods are used to design the filter coefficients again and again until a particular error is minimized. The various methods are as follows:

i. Least square method.
ii. Equiripple method.
iii. Maximally flat.
iv. Generalised equiripple.
v. Constrained band equiripple.

In Least square method there is no constraint on the response between the sample points, and poor results may be obtained. The frequency sampling technique is more of an
interpolation method rather than an approximation method. Least square method [6-7] controls the response between the sample points by considering a number of sample points larger than the order of the filter. As the energy of the signal is related to the square of the signal, a squared error approximation criterion is appropriate to optimize the design of the FIR filters.

The Remez/Parks McLellan method produces a filter which just meets the specification without overperforming. Many of the window method designs actually perform better as you move further away from the passband, and means they are using more filter coefficients than they need. This means that they are using more filter coefficients than they need.

To design equiripple linear-phase FIR filters with linear constraint method is carried out by using the Remez exchanges algorithm. A technique is derived to convert a linearly constrained problem to an equivalent unconstrained one. The key step is to modify the original desired frequency response such that the constraint is reduced to a null constraint for the new target frequency response. Then the filter constrained by such constraint can be designed without any constraint by a set of bases obtained by transforming the original basis by the null space. [8]

Equiripple design produces most efficient filter with minimum possible order.

**Table 3** Optimal Filter Methods for digital FIR filter design

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>FIR filter design Technique</th>
<th>Passband attenuation($\delta_p$)</th>
<th>Sopband attenuation($\delta_s$)</th>
<th>Order of Window(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equiripple</td>
<td>1.91601 dB</td>
<td>60.9234 dB</td>
<td>71</td>
</tr>
<tr>
<td>2</td>
<td>Least square</td>
<td>0.01939 dB</td>
<td>60.45124 dB</td>
<td>185</td>
</tr>
<tr>
<td>3</td>
<td>Maximally flat</td>
<td>1.2554 dB</td>
<td>4.56 dB</td>
<td>110</td>
</tr>
<tr>
<td>4</td>
<td>Generalized equiripple</td>
<td>2.63329 dB</td>
<td>60.496 dB</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>Constrained band equiripple</td>
<td>0.08426 dB</td>
<td>60.6305 dB</td>
<td>145</td>
</tr>
</tbody>
</table>

**VI. Conclusion**
The paper briefly describes the various technique used to design FIR filter. The major advantage of window technique is its simplicity. The availability of well defined equations for calculating window coefficient has made this method preferable. But it offers very little design flexibility especially in low pass filter design. Kaiser window offers very low order to meet given specification. The best digital filter design results comes for using the Kaiser window from the windowing design technique, which has parameter $\beta$ that allows adjustment of the compromise between the overshoot reduction and transition region width spreading. But when Kaiser window is compared with optimal filter design method, equiripple filter design found to be most suitable and optimized method to meet given specification.

References


