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ON UNIVEXITY-TYPE NONLINEAR PROGRAMMING PROBLEMS IN COMPLEX SPACES

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Abstract :

In this paper , we will study a new class of nonlinear programming called SFJ-univex introduced in [13] in complex spaces , combining the concepts of SFJ-invex programming and univex functions in complex spaces. Optimality and duality results for several mathematical programs are obtained under the above –mentioned assumption .

1.Introduction :

Consider the following nonlinear programming problem :

(p) $\min \ \mathrm{Re}\, f[z, \bar{z}, w, \overline{w}]$

such that $\operatorname{Re} g[z,\bar{z},w,\overline{w}] \le 0$, $[z,\bar{z},w,\overline{w}] \in$, $C^n \times C^n \times C^m \times C^m$

where $f : C^n \times C^n \times C^m \times C^m \to C$, $g : C^n \times C^n \times C^m \times C^m \to C$.

Several classes of functions have been defined for the purpose of weakening the limitations of convexity in the mathematical programming problem (P).

Xu[16] proposed the new class of nonlinear programming , called SFJ-invex programming,which lies between invex programming and type I programming [3,6,7].

Bector et al. [2] introduced the concept of univexity as a generalization of convexity. Recently , Rueda et al. [10] introduced a new class of functions , combining the concepts of type I and univex functions and obtained optimality and duality results for several mathematical programs .Then further S.K.Mishra and N.G.Rueda [13] introduced and discussed SFJ-univex programming problems .This paper can be view as extension of [13] in complex spaces.

Again after enhancement of these area after introduction in complex spaces these area become wider then a lot of consequence of papers have been published yet , some of them are [1],[4],[5],[8],[11],[14],[15].

2. Preliminaries:

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In this paper , we introduced the SFJ-univex functions in complex spaces and proposed the following .

To compare vectors along the lines of Mangasarian , we will distinguish between $\leq$ and $\leqq$ or $\geq$ and $\geqq$, specifically ,in complex space . Let For C^n denote an n-dimensional complex spaces .For $z \in C^n$, let the real vectors Re (z) and Im (z) denote the real and imaginary parts , respectively , and let $\bar{z} = \text{Re}(z) - i\text{Im}(z)$ be the conjugate of z . Given a matrix $A = \lfloor a_{ij} \rfloor \in C^{m \times n}$, where $C^{m \times n}$ is the collection of m×n complex matrices , let $\overline{A} = \lfloor \overline{a}_{ij} \rfloor \in C^{m \times n}$ denote its conjugate matrix , let $A^H = \lfloor \overline{a}_{ij} \rfloor$ denote its conjugate transpose . The inner product of $x, y \in C^n$ is $(x, y) = y^H x$ .Let R_+ denote the half line $[0,\infty[$.

$z \in C^n$, $v \in C^n$, $\text{Re}(z) \leqq \text{Re}(v) \Leftrightarrow \text{Re}(z_i) \leqq \text{Re}(v_i)$, for all i = 1,2,.,.,.,n , $\text{Re}(z) \neq \text{Re}(v)$.

Similar notations are applied to distinguish between $\geq$ and $\geqq$.

For a complex function $f : C^n \times C^n \times C^m \times C^m \to C$ analytic with respect to $\zeta = (w^1, w^2)$, $z \in C^n$, define gradients by

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$$\nabla_z f_i(v,\bar{v},\zeta)=[\frac{\partial f}{\partial w_i^1}(v,\bar{v},\zeta)] \quad i=1,2,.,.,.,n\ .$$

$$\nabla_{\bar{z}} f_i(v,\bar{v},\zeta)=[\frac{\partial f}{\partial w_i^2}(v,\bar{v},\zeta)] \quad i=1,2,.,.,.,n\ .$$

Definition 2.1:

The Problem (P) is said to be SFJ-univex if there exist $\eta : C^n \times C^n \times C^m \times C^m \to C$,

$\phi_0 : C \to C$, $\phi_i : C \to C, i=1,2,....,m$, $b_0 : C^n \times C^n \times C^m \times C^m \to C_+$,

$b_i : C^n \times C^n \times C^m \times C^m \to C_+$, i=1,2,.,.,.,k .

Such that

$$\left.\begin{array}{l}(z_0,\bar{z}_0)\in C^n\times C^n\\ \\ \\ (z,\bar{z})\in C^n\times C^n\end{array}\right\}\left\{\begin{array}{l}\mathrm{Re}[b_0(z,\bar{z},z_0,\bar{z}_0)\{\varphi_0\{f(z,z,w,\bar{w})-f(z_0,\bar{z}_0,w,\bar{w})\}]\\ \geq \mathrm{Re}[\overline{\nabla_z f(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)}\\ +\nabla_{\bar{z}} f(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)]\\ \\ \text{If } \mathrm{Re}[g_i(z,z,w,\bar{w})]=0, i=1,2,.,.,.,k.\text{ then}\end{array}\right. \quad (1)$$

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$$\begin{aligned}&\operatorname{Re}[b_i(z,\bar{z},z_0,\bar{z}_0)\{\varphi_i\{g_i(z,z,w,\bar{w})\}\}]\\ &\geq \operatorname{Re}[\overline{\nabla_z g(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)}\\ &+\nabla_{\bar{z}} g(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)]\end{aligned} \tag{2}$$

Definition 2.2:

The Problem (P) is said to be SFJ-invex if there exist $\eta : C^n \times C^n \times C^m \times C^m \to C$, Such that

$$\left.\begin{array}{l}(z_0,\bar{z}_0) \in C^n \times C^n \\ \\ (z,\bar{z}) \in C^n \times C^n\end{array}\right\} \Rightarrow \begin{cases}\operatorname{Re}[f(z,z,w,\bar{w}) - f(z_0,\bar{z}_0,w,\bar{w})] \\ \geq \operatorname{Re}[\overline{\nabla_z f(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)} \\ +\nabla_{\bar{z}} f(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)] & (3) \\ \text{If } \operatorname{Re}[g_i(z,z,w,\bar{w})] = 0, i = 1,2,.,.,.,k. \text{ then} \\ \quad \operatorname{Re}[g_i(z,z,w,\bar{w})] \\ \quad \geq \operatorname{Re}[\overline{\nabla_z g_i(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)} \\ \quad +\nabla_{\bar{z}} g_i(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)] & (4)\end{cases}$$

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Theorem 2.1:

Every SFJ-invex program is an SFJ-univex program , but the converse may not be true .

Proof:

Assume that a given program (P) is SFJ-invex , i.e., there exists a function

$\eta : C^n \times C^n \times C^m \times C^m \to C$, Such that

$$\left.\begin{array}{l} (z_0, \bar{z}_0) \in C^n \times C^n \\ (z, \bar{z}) \in C^n \times C^n \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \mathrm{Re}[f(z,z,w,\bar{w}) - f(z_0,\bar{z}_0,w,\bar{w})] \\ \geq \mathrm{Re}[\overline{\nabla_z f(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)} \\ +\nabla_{\bar{z}} f(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)] \\ \text{If } \mathrm{Re}[g_i(z,z,w,\bar{w})] = 0, i = 1,2,.,.,.,k. \text{ then} \\ \mathrm{Re}[g_i(z,z,w,\bar{w})] \\ \geq \mathrm{Re}[\overline{\nabla_z g_i(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)} \\ +\nabla_{\bar{z}} g_i(z_0,\bar{z}_0,w,\bar{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)] \end{array}\right.$$

Hence , problem (P) is SFJ-univex with respect to $b_0 = b_i = 1$, $\mathrm{Re}[\phi_0(f)] = \mathrm{Re}[f]$, $\mathrm{Re}[\phi_i(g)] = \mathrm{Re}[g_i]$, for i=1,2,.,.,.,k and the same η .

For the converse part see the following example :

Example 2.1:

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Consider the following problem :

$\min \; f[z,\bar{z},w,\overline{w}] = z+\bar{z}$ such that $g[z,\bar{z},w,\overline{w}] = -[z+\bar{z}]+1 \le 0$

The problem is SFJ-univex with respect to $\eta[z,\bar{z},z_0,\bar{z}_0] = \frac{1}{2}[z-z_0+\bar{z}-\bar{z}_0]$,

$b_0 = b_1 = 1, \phi_0(f) = f$, $\phi_1 = -g$, at $z_0 = 1$, but the problem is not SFJ-invex at $z_0 = 1$ since $g[z_0,\bar{z}_0,w,\overline{w}] = 0$ but

$$\mathrm{Re}[g_i(z,z,w,\overline{w})]$$
$$\ge \mathrm{Re}[\overline{\nabla_z g_i(z_0,\bar{z}_0,w,\overline{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)} + \nabla_{\bar{z}} g_i(z_0,\bar{z}_0,w,\overline{w})\eta^T(z,\bar{z},z_0,\bar{z}_0)] =$$

$$-[z+\bar{z}]+1-\{\overline{\frac{1}{2}[z-1+\bar{z}-1][-1]} + \frac{1}{2}[z-1+\bar{z}-1][-1]\} = -[z+\bar{z}]+1+[z+\bar{z}]-2 = -1 \le 0$$

So the problem is SFJ-univex but not SFJ-invex .

3. Nonlinear programming :

In this section we will show that optimality and duality results still hold for the nonlinear problem (P) under weaker generalized convexity conditions.

Theorem 3.1:Optimality :

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Let $(z_0, \bar{z}_0, w, \overline{w})$ be (P) feasible for the SFJ-univex problem (P) .Suppose that there

$(u, \bar{u}, w, \overline{w}) \in \mathrm{C}^n \times \mathrm{C}^n \times \mathrm{C}^m \times \mathrm{C}^m$

subject to

$$\mathrm{Re}[\overline{\nabla_z f[z_0, \bar{z}_0, w, \overline{w}]} + \nabla_{\bar{z}} f[z_0, \bar{z}_0, w, \overline{w}]\} + u^T \overline{\nabla_z g[z_0, \bar{z}_0, w, \overline{w}]} + u^H \nabla_{\bar{z}} g[z_0, \bar{z}_0, w, \overline{w}]\}] = 0 \quad (5)$$

$$\mathrm{Re}[g_i[z_0, \bar{z}_0, w, \overline{w}]] < 0 \Rightarrow u_i = 0, \bar{u}_i = 0, i = 1,2,.,.,k \quad (6)$$

and $u \geq 0,$ (7)

Further suppose that $\mathrm{Re}[\phi_0(f_1)] \geq 0 \Rightarrow \mathrm{Re}[f_1] \geq 0$ (8)

$$\mathrm{Re}[f_1] \leq 0 \Rightarrow \mathrm{Re}[\phi_0(f_1)] \leq 0 \quad (9)$$

and that $\mathrm{Re}[b_0(z, \bar{z}, z_0, \bar{z}_0)] > 0$ (10)

for all feasible $(z, z, w, \overline{w})$. Then $(z_0, \bar{z}_0, w, \overline{w})$ is an optimal solution of (P).

Proof :

Let $(z, z, w, \overline{w})$ be (P) feasible . Then (1),(2),(5),(6),(7),(8),and (9) , we have

$$\mathrm{Re}[b_0(z, \bar{z}, z_0, \bar{z}_0)\{\phi_0\{f(z, z, w, \overline{w}) - f(z_0, \bar{z}_0, w, \overline{w})\}]$$
$$\geq \mathrm{Re}[\overline{\nabla_z f(z_0, \bar{z}_0, w, \overline{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)} + \nabla_{\bar{z}} f(z_0, \bar{z}_0, w, \overline{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)]$$

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$$= \operatorname{Re}[\sum_{i=1}^{k} u_i \overline{\nabla_z g_i(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)} + \sum_{i=1}^{k} \bar{u}_i \nabla_{\bar{z}} g_i(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)]$$

$$\geq -\operatorname{Re}[\sum_{i=1}^{k} b_i(z, \bar{z}, z_0, \bar{z}_0)(u, \bar{u}, w, \bar{w})\{\phi_i\{g_i(z, z, w, \bar{w})\}\}] \geq 0 .$$

From (10) , it follows that

$$\operatorname{Re}\{\phi_0\{f(z, \bar{z}, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w})\} \geq 0$$

By (8) , we have $\operatorname{Re}[f(z, z, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w})] \geq 0$

Therefore , $(z_0, \bar{z}_0, w, \bar{w})$ is an optimal solution of (P).

(D) Max. $\operatorname{Re}[f(v, \bar{v}, w, \bar{w})$

Subject to

$$\operatorname{Re}[\{\overline{\nabla_z f[v, \bar{v}, w, \bar{w}]} + \nabla_{\bar{z}} f[v, \bar{v}, w, \bar{w}]\} + x^T \overline{\nabla_z g[v, \bar{v}, w, \bar{w}]} + x^H \nabla_{\bar{z}} g[v, \bar{v}, w, \bar{w}]\}] = 0$$

$$\operatorname{Re}\{[y + \bar{y}]^T g[v, \bar{v}, w, \bar{w}]\} \geq 0, \ y \geq 0$$

Duality results can be obtained under similar conditions . We show some of them below.

Theorem 3.2 (weak duality):

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Let $(x,\bar{x},w,\bar{w})$ be (P)-feasible and $\{(u,\bar{u},w,\bar{w}),y\}$ be (D)-feasible . If there exist $\eta,\phi_0,b_0,\phi_i,b_i,i=1,2,.,.,.m$, with ϕ_0 strictly increasing , such that conditions (1),(2) and (8) are satisfied at $((x,\bar{x},w,\bar{w}),(u,\bar{u},w,\bar{w}))$, $y_i=0$ when $\text{Re}[g_i(u,\bar{u},w,\bar{w})]>0$, and $\sum_{i=1}^{m} b_i(x,\bar{x},u,\bar{u})(y_i+\bar{y}_i)\phi_i(g_i(x,\bar{x},w,\bar{w}))\le 0$, then $\text{Re}[f(x,\bar{x},w,\bar{w})-f(u,\bar{u},w,\bar{w})]\ge 0$.

Proof:

Assume that $\text{Re}[f(x,\bar{x},w,\bar{w})-f(u,\bar{u},w,\bar{w})]<0$. Then

$$\text{Re}[b_0(x,\bar{x},u,\bar{u})\phi_0\{f(x,\bar{x},w,\bar{w})-f(u,\bar{u},w,\bar{w})\}]<0 \qquad (11)$$

for all feasible ($x,\bar{x},,u,\bar{u},y$).

On the other hand , by the definition of SFJ-Univexity and the assumptions of the theorem , we have

$$\text{Re}[b_0(x,\bar{x},u,\bar{u})\phi_0\{f(x,\bar{x},w,\bar{w})-f(u,\bar{u},w,\bar{w})\}]$$

$$\ge \text{Re}[\overline{\eta^T(x,\bar{x},u,\bar{u})\nabla_x f(u,\bar{u},w,\bar{w})}+\eta^T(x,\bar{x},u,\bar{u})\nabla_{\bar{x}} f(u,\bar{u},w,\bar{w})]$$

$$= \text{Re}[\overline{\eta^T(x,\bar{x},u,\bar{u})\sum_{i=1}^{m} y_i^T\nabla_x g_i(u,\bar{u},w,\bar{w})}+\eta^T(x,\bar{x},u,\bar{u})\sum_{i=1}^{m} y_i^H\nabla_{\bar{x}} g_i(u,\bar{u},w,\bar{w})]$$

$$\ge -\sum_{i=1}^{m} b_i(x,\bar{x},u,\bar{u})(y_i+\bar{y}_i)\phi_i(g_i(x,\bar{x},w,\bar{w}))\ge 0$$, which contradicts (9). Hence the result .

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Theorem 3.3 (Strong duality):

If $(x_*, \bar{x}_*, w, \overline{w})$ is (P)-feasible and a constraint qualification is satisfied at $(x_*, \bar{x}_*, w, \overline{w})$, then there exists $y_* \in C^m$ such that $(x_*, \bar{x}_*, y_*)$ is (D)-feasible and the values of the objective functions for (P) and (D) are equal at $(x_*, \bar{x}_*, w, \overline{w})$ and $(x_*, \bar{x}_*, y_*)$, respectively .Furthermore, if for all (P)-feasible $(x, \bar{x}, w, \overline{w})$ and (D)-feasible $(u, \bar{u}, y)$, the hypothesis of the theorem 3.2 are satisfied , then $(x_*, \bar{x}_*, y_*)$ is (D)-optimal .

Proof:

Since a constraint qualification is satisfied at $(x_*, \bar{x}_*, w, \overline{w})$ then there exists $y_* \in C^m$ such that the following Kuhn-Tucker conditions are satisfied .

$$\mathrm{Re}[\overline{\nabla_{u} f(x_*, \bar{x}_*, w, \overline{w})} + y_*^T \overline{\nabla_{u} g(x_*, \bar{x}_*, w, \overline{w})} + \nabla_{\bar{u}} f(x_*, \bar{x}_*, w, \overline{w}) + y_*^H \nabla_{\bar{u}} g(x_*, \bar{x}_*, w, \overline{w})] = 0$$
$$(y_*^T + y_*^H) g(x_*, \bar{x}_*, w, \overline{w}) = 0, y_* \geq 0$$

Therefore $(x_*, \bar{x}_*, y_*)$ is (D)-feasible .

Suppose that $(x_*, \bar{x}_*, y_*)$ is not (D)-optimal . Then there exists (D)-feasible $(u, \bar{u}, y)$ such that

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$\mathrm{Re}[f(x_*,\bar{x}_*,w,\bar{w})-f(u,\bar{u},w,\bar{w})]$<0. This contradicts theorem 3.2 .

Therefore $(x_*,\bar{x}_*,y_*)$ is (D)-optimal .

4. Multiobjective Programming :

Consider the following problems :

(MP) Min $\mathrm{Re}[f_i(x,\bar{x},w,\bar{w})]$ such that $\mathrm{Re}[g_J(x,\bar{x},w,\bar{w})]\le 0$, where

$f_i : C^n \times C^n \times C^m \times C^m \to C, i=1,2,.,.,.,p$, and $g_j : C^n \times C^n \times C^m \times C^m \to C, j=1,2,.,.,.,k$,

m,n<p,k , f_i and g_j are all differentiable functions and the minimum is obtained in terms of efficiency as defined below;

(MD) Max. $\mathrm{Re}[f_i(u,\bar{u},w,\bar{w})]$

subject to

$$\mathrm{Re}[\nabla_x \overline{\{v_i^T f(u,\bar{u},w,\bar{w})+y_j^T g(u,\bar{u},w,\bar{w})\}}+\nabla_{\bar{x}}\{v_i^H f(u,\bar{u},w,\bar{w})+y_j^H g(u,\bar{u},w,\bar{w})\}]=0$$

$$\mathrm{Re}[(y_i^T + y_j^H)g(u,\bar{u},w,\bar{w})]\ge 0 \quad v\ge 0, y\ge 0 .$$

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Definition 4.1: An (MP)-feasible $(x_*, \bar{x}_*, w, \bar{w})$ is said to be an efficient solution of (MP) if there exists no $(x, \bar{x}, w, \bar{w})$ such that $\operatorname{Re}[f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w})] \leq 0$.

Now, we will establish optimality conditions for a point to be an efficient solution of (MP).

Theorem 4.1 (Optimality):

Let $(x_*, \bar{x}_*, w, \bar{w})$ be (MP)-feasible , suppose that there exist $v_* \in C^p$, $y_* \in C^k$, η, b_0, b_1, ϕ_0 and ϕ_1 such that

$$\begin{aligned}&\operatorname{Re}[b_0(x, \bar{x}, x_*, \bar{x}_*)\phi_0\{v_*^T(f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w}))\}] \\ &\geq \operatorname{Re}[\eta^T(x, \bar{x}, x_*, \bar{x}_*)\{v_*^T \overline{\nabla_x f(x_*, \bar{x}_*, w, \bar{w})} + v_*^H \nabla_{\bar{x}} f(x_*, \bar{x}_*, w, \bar{w})\}]\end{aligned} \tag{12}$$

$$\begin{aligned}&\operatorname{Re}[b_1(x, \bar{x}, x_*, \bar{x}_*)\phi_1\{y_*^T g(x, \bar{x}, w, \bar{w})\}] \\ &\geq \operatorname{Re}[\eta^T(x, \bar{x}, x_*, \bar{x}_*)\{y_*^T \overline{\nabla_x g(x_*, \bar{x}_*, w, \bar{w})} + y_*^H \nabla_{\bar{x}} g(x_*, \bar{x}_*, w, \bar{w})\}]\end{aligned} \tag{13}$$

for all (MP)-feasible $(x, \bar{x}, w, \bar{w})$ and

$$\begin{aligned}&\operatorname{Re}[\nabla_x \overline{\{v_*^T f(u, \bar{u}, w, \bar{w}) + y_*^T g(u, \bar{u}, w, \bar{w})\}} \\ &+\nabla_{\bar{x}}\{v_*^H f(u, \bar{u}, w, \bar{w}) + y_*^H g(u, \bar{u}, w, \bar{w})\}]\end{aligned} \tag{14}$$

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$v_* \geq 0, y_* \geq 0$ (15)

further suppose that $f \leq 0 \Rightarrow \phi_0(f)$ (16)

$$\phi_1(f) \geq 0 \Rightarrow f > 0 \quad (17)$$

$$b_0(x,\bar{x},x_*,\bar{x}_*) \geq 0, b_1(x,\bar{x},x_*,\bar{x}_*) > 0 \quad (18)$$

for all feasible $(x,\bar{x},w,\overline{w})$ .Then $(x_*,\bar{x}_*,w,\overline{w})$ is an efficient solution of (MP).

Proof:

Let $(x,\bar{x},w,\overline{w})$ be (MP)-feasible ,suppose that $\operatorname{Re}[f(x,\bar{x},w,\overline{w}) - f(x_*,\bar{x}_*,w,\overline{w})] \leq 0$. Then

$\operatorname{Re}[v_*^T\{f(x,\bar{x},w,\overline{w}) - f(x_*,\bar{x}_*,w,\overline{w})\}] \leq 0$ i.e.,

Fro (14) and (16) it follows that

$\operatorname{Re}[b_0(x,\bar{x},x_*,\bar{x}_*)\phi_0\{v_*^T(f(x,\bar{x},w,\overline{w}) - f(x_*,\bar{x}_*,w,\overline{w}))\}] \leq 0,$

therefore by (12) $\operatorname{Re}[\eta^T(x,\bar{x},x_*,\bar{x}_*)\{v_*^T\overline{\nabla_x f(x_*,\bar{x}_*,w,\overline{w})} + v_*^H\nabla_{\bar{x}} f(x_*,\bar{x}_*,w,\overline{w})\}] \leq 0,$

then by (14) $\operatorname{Re}[\eta^T(x,\bar{x},x_*,\bar{x}_*)\{y_*^T\overline{\nabla_x g(x_*,\bar{x}_*,w,\overline{w})} + y_*^H\nabla_{\bar{x}} g(x_*,\bar{x}_*,w,\overline{w})\}] \geq 0,$

from (13) ,we obtain

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$\mathrm{Re}[b_1(x,\bar{x},x_*,\bar{x}_*)\phi_1\{(y_*^T+y_*^H)g(x,\bar{x},w,\overline{w})\}]\geq 0$. By (17) and (18) it follows that

$(y_*^T+y_*^H)g(x,\bar{x},w,\overline{w})>0$, which is a contradiction to (13) and the (MP)-feasibility of $(x,\bar{x},w,\overline{w})$. Therefore $(x_*,\bar{x}_*,w,\overline{w})$ is an efficient solution of (MP).

Theorem 4.2 (Weak duality):

Let $(x,\bar{x},w,\overline{w})$ be (MP)-feasible and $\{(u,\bar{u},w,\overline{w}),y\}$ be (MD)-feasible . If there exist η, b_0, b_1, ϕ_0 and ϕ_1 such that

$$\mathrm{Re}[b_0(x,\bar{x},u,\bar{u})\phi_0\{v_*^T(f(x,\bar{x},w,\overline{w})-f(u,\bar{u},w,\overline{w}))\}]$$
$$\geq \mathrm{Re}[\eta^T(x,\bar{x},u,\bar{u})\{v_*^T\overline{\nabla_x f(u,\bar{u},w,\overline{w})}+v_*^H\nabla_{\bar{x}}f(u,\bar{u},w,\overline{w})\}]$$

$$\mathrm{Re}[b_1(x,\bar{x},u,\bar{u})\phi_1\{y_*^T g(x,\bar{x},w,\overline{w})\}]$$
$$\geq \mathrm{Re}[\eta^T(x,\bar{x},u,\bar{u})\{y_*^T\overline{\nabla_x g(u,\bar{u},w,\overline{w})}+y_*^H\nabla_{\bar{x}}g(u,\bar{u},w,\overline{w})\}]$$

(16) and (17) hold and (18) is satisfied at $((x,\bar{x},w,\overline{w}),(u,\bar{u},w,\overline{w}))$ then $\mathrm{Re}[f(x,\bar{x},w,\overline{w})-f(u,\bar{u},w,\overline{w})]>0$.

Proof:

The proof is similar to that of Theorem 4.1.

Theorem 4.3(Strong duality):

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If $(x_*, \bar{x}_*, w, \overline{w})$ is an efficient solution of (MP)and a suitable constraint qualification holds at $(x_*, \bar{x}_*, w, \overline{w})$ as Singh's[3] and analogous with Rueda and Mishra [13] , then there exist v_* and y_* such that $((x_*, \bar{x}_*, w, \overline{w}), v_*, y_*)$ is feasible for the dual (MD) and the values of the objective functions for (MP) and (MD) are equal at $(x_*, \bar{x}_*, w, \overline{w})$ and $((x_*, \bar{x}_*, w, \overline{w}), v_*, y_*)$,respectively. Furthermore , if $v_*^T f$ and $y_*^T g$ satisfy conditions (12) and (13) , ϕ_0 and ϕ_1 satisfy (16) and (17), and b_0 and b_1 satisfy (18) then $((x_*, \bar{x}_*, w, \overline{w}), v_*, y_*)$ is efficient for (MD).

Proof:

From the assumption it follows that $((x_*, \bar{x}_*, w, \overline{w}), v_*, y_*)$ is (MD)-feasible . Suppose that it is not efficient . Then there exists $((u, \bar{u}, w, \overline{w}), v, y)$ feasible such that $\text{Re}[f(x, \bar{x}, w, \overline{w}) - f(x_*, \bar{x}_*, w, \overline{w})] \geq 0$.

This contradicts Theorem 4.2 . Hence , $((x_*, \bar{x}_*, w, \overline{w}), v_*, y_*)$ is efficient for (MD).

5. Conclusion:

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In this paper , we have extended an earlier work of Rueda et al. [10] to SFJ-univexity conditions .Results for minmax programming and generalized fractional programming problems can be obtained on similar lines .

Extension of this work under SFJ-pseudo-univexity in real spaces and other conditions would extend anearlier work of Kaul et al.[12]. It also extends the work of [13] .

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